# Design and Analysis of Cooperative Task-Processing Agents

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**Abstract**

*This work describes the dynamical behavior of spatially constrained agents that can each share the task-processing burden of their immediate neighbors. The work is influenced by studies of the evolution of cooperation and extends existing work in the design of resource-allocation strategies on cooperative agents. A framework for shared task processing on a network is presented, and theoretical results show sufficient conditions on distributed and asynchronous agent behaviors that guarantee an optimal allocation of task-processing resources on the network. These sufficient conditions include a network version of Hamilton's rule. In particular, in order to guarantee convergence of the gradient ascent algorithm on each agent, the benefit-to-cost ratio of cooperation must be greater than a measure of the topological relatedness between agents. The framework is shown to be applicable for autonomous air vehicles (AAV), mobile software agents, and smart power grids, and simulation results are given for an AAV case. However, it may also aid in the modeling and analysis of biological (e.g., cooperative breeding) and social systems where cooperation is not perceived to be mutually beneficial and cannot be enforced by fiat.*

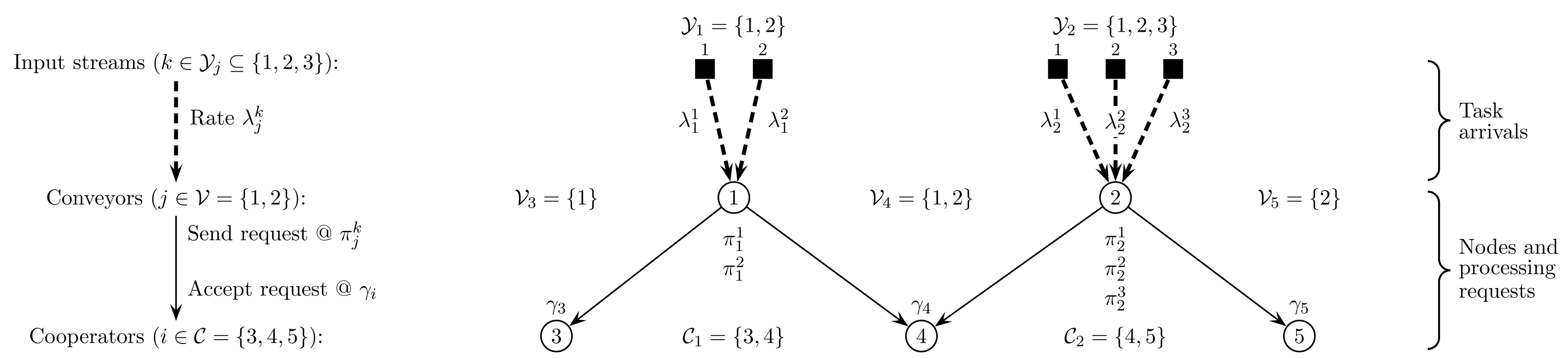
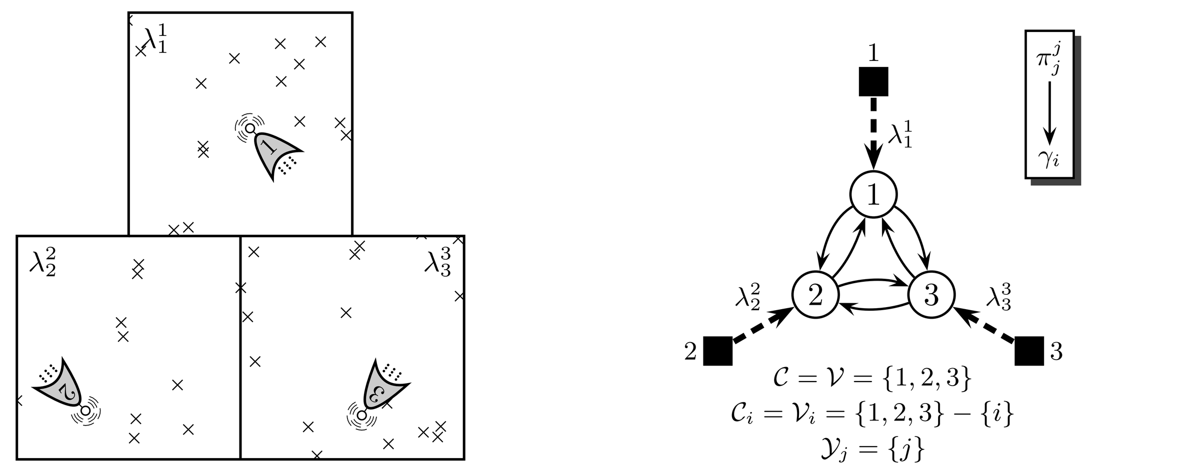



**Fig. 1:** *Sample TPN: A flexible manufacturing system (FMS).*




( a ) AAV patrol scenario. ( b ) AAV task-processing network.

**Fig. 2:** *Sample TPN: Autonomous air vehicle (AAV) task scheduling.*

## Task-processing network

- $\mathcal{A} \subset \mathbb{N}$: Set of *task-processing agents*
- $\mathcal{P} \subseteq \{(i,j) \in \mathcal{A}^2 : i \neq j\}$: Directed arcs connecting distinct agents
- $\mathcal{V}_i \triangleq \{j \in \mathcal{A} : (j,i) \in \mathcal{P}\}$: Set of *conveyors* for each $i \in \mathcal{A}$
- $\mathcal{C}_i \triangleq \{j \in \mathcal{A} : (i,j) \in \mathcal{P}\}$: Set of *cooperators* for each $i \in \mathcal{A}$
- $\mathcal{V} \triangleq \{j \in \mathcal{A} : \mathcal{C}_j \neq \emptyset\}$: Set of all conveyors
- $\mathcal{C} \triangleq \{i \in \mathcal{A} : \mathcal{V}_i \neq \emptyset\}$: Set of all cooperators
- $\mathcal{Y}_i \subset \mathbb{N}$: Possibly empty set of *task types* that arrive at conveyor $i \in \mathcal{A}$
- $\lambda_j^k \in \mathbb{R}_{>0}$: Encounter rate of type-$k$ tasks at agent $j \in \mathcal{A}$
- $\pi_j^k \in [0,1]$: Probability that conveyor $j \in \mathcal{A}$ advertises an incoming $k$-type task to its connected cooperators $\mathcal{C}_j$
- $\gamma_i \in [0,1]$: Probability that cooperator $i \in \mathcal{A}$ volunteers for advertised task from one of its connected conveyors $\mathcal{V}_i$

## Agent utility function

$$U_i(\underline{\gamma}) \triangleq \overbrace{b_i + \underbrace{\left(1 - \prod_{j \in \mathcal{C}_i} (1-\gamma_j)\right)}_{\Pr(\text{Volunteer from } \mathcal{C}_i | \text{Advertisement from } i)} r_i - Q_i p_i(Q_i)}^{\text{Conveyor part — constant with respect to } \gamma_i}$$

$$\underbrace{+ \gamma_i \sum_{j \in \mathcal{V}_i} \left(p_{ij}(Q_j) - \overbrace{\mathrm{SOBP}_1(\mathcal{C}_j - \{i\})}^{\Pr(i \text{ awarded task from } j | i \text{ volunteers})} c_{ij}\right)}_{\text{Cooperator part — } \gamma_i \text{ and } Q_j \text{ vary with } \gamma_i}$$

where

$$b_i \triangleq \sum_{k \in \mathcal{Y}_i} \lambda_i^k \left(b_i^k - c_i^k\right),$$

$$r_i \triangleq \sum_{k \in \mathcal{Y}_i} \lambda_i^k \pi_i^k \left(r_i^k - \left(b_i^k - c_i^k\right)\right),$$

$$p_i(Q_i) \triangleq \sum_{k \in \mathcal{Y}_i} \lambda_i^k \pi_i^k p_i^k(Q_i),$$

are the costs and benefits of local processing on $i \in \mathcal{V}$, and

$$c_{ij} \triangleq \sum_{k \in \mathcal{Y}_j} \lambda_j^k \pi_j^k c_{ij}^k,$$

$$p_{ij}(Q_j) \triangleq \sum_{k \in \mathcal{Y}_j} \lambda_j^k \pi_j^k q_{ij}^k p_j^k(Q_j).$$

are the costs and benefits to $i \in \mathcal{C}$ for volunteering for tasks exported from $j \in \mathcal{V}_i$.

## Asynchronous convergence to cooperation

Assume that (Payment and topological constraints)

1. For all $i \in \mathcal{C}$ and $j \in \mathcal{V}_i$, $p_{ij}$ is a stabilizing payment function.
2. For all $j \in \mathcal{V}$, $|\mathcal{C}_j| \leq 3$ (i.e., no conveyor can have more than 3 outgoing links to cooperators).
3. For $i \in \mathcal{C}$ and $j \in \mathcal{V}_i$, if $j$ is a 3-conveyor, then there must be some $k \in \mathcal{V}_i$ that is a 2-conveyor.

Define $T : [0,1]^n \mapsto [0,1]^n$ by $T(\underline{\gamma}) \triangleq (T_1(\underline{\gamma}), T_2(\underline{\gamma}), \dots, T_n(\underline{\gamma}))$ where, for each $i \in \mathcal{C}$,

$$T_i(\underline{\gamma}) \triangleq \min\{1, \max\{0, \gamma_i + \sigma_i \nabla_i U_i(\underline{\gamma})\}\}$$

(i.e., gradient ascent), where

$$\frac{1}{\sigma_i} \geq 2|\mathcal{V}_i| \max_{k \in \mathcal{V}_i} |p'_{ik}(0)|$$

for all $\underline{\gamma} \in [0,1]^n$. If (Hamilton's rule on networks)

$$\overbrace{\min_{j \in \mathcal{V}_i} |p'_{ij}\left(|\mathcal{C}_j|\right)|}^{\text{Benefit}} > \overbrace{\left(|\mathcal{V}_i| - \frac{1}{2}\right)}^{\text{Relatedness}} \overbrace{\max_{j \in \mathcal{V}_i} |c_{ij}|}^{\text{Cost}}, \quad \text{for all } i \in \mathcal{C},$$

then the totally asynchronous distributed iteration (TADI) sequence $\{\underline{\gamma}(t)\}$ generated with mapping $T$ and the outdated estimate sequence $\{\underline{\gamma}^i(t)\}$ for all $i \in \mathcal{C}$ each converge to the unique Nash equilibrium of the cooperation game.

## Stabilizing payment functions

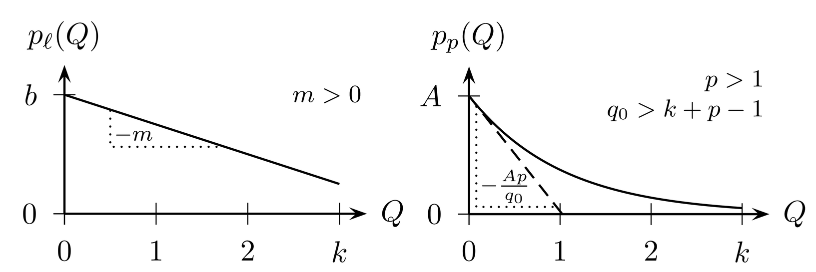



**Fig. 3:** *Sample stabilizing payment functions*

## Simulation results

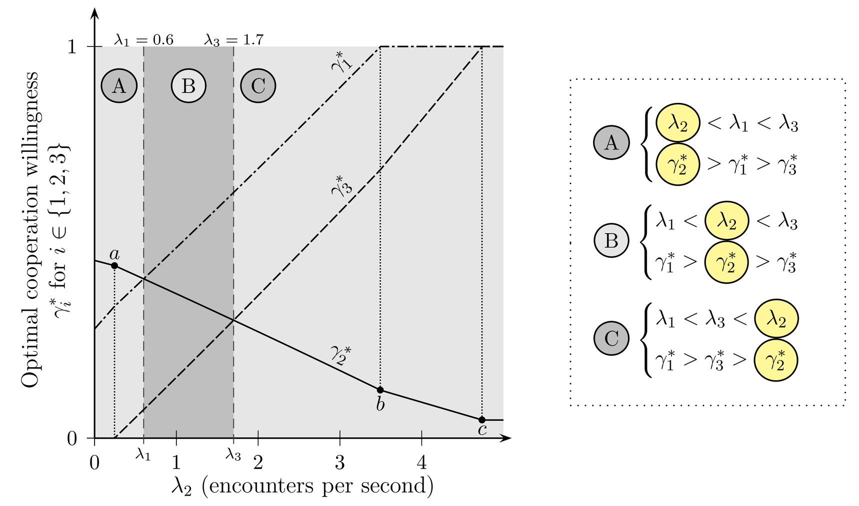



**Fig. 4:** *Simulation of AAV patrol scenario shows convergence to predicted Nash. Increases in one encounter rate cause equilibrium shift so neighbors help more and agent helps less.*

## Other stable topologies

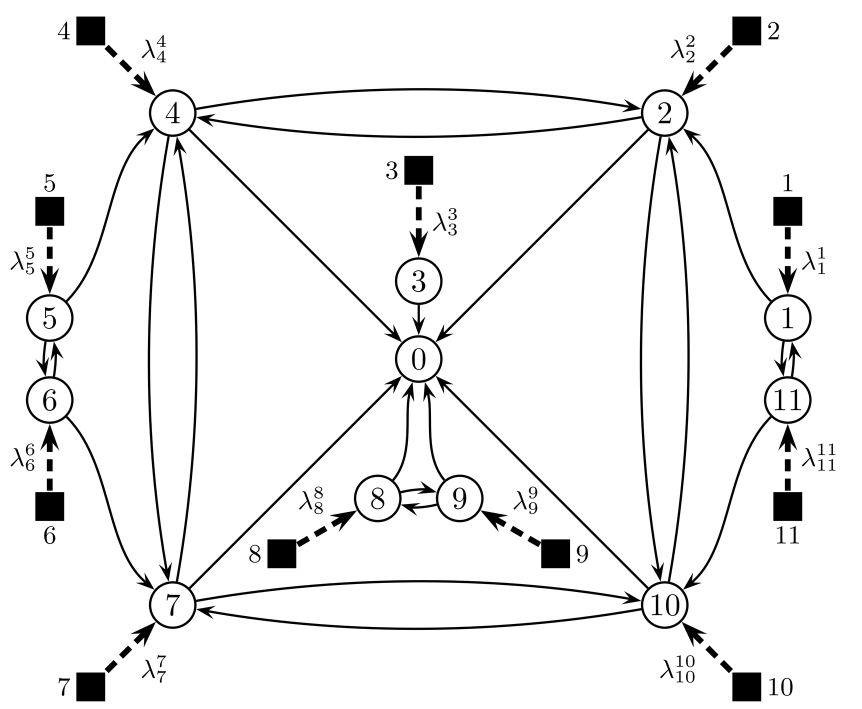


**Fig. 5:** *Rich yet stable task-processing network. "Pills" stabilize problematic areas by focussing attention.*

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