

Cooperative Task Processing

Theodore P. Pavlic and Kevin M. Passino
Department of Electrical & Computer Engineering
The Ohio State University
pavlic.3@osu.edu and passino@ece.osu.edu



Abstract

This work describes the dynamical behavior of agents that can each share the task-processing burden of their immediate neighbors. The work is influenced by studies of the evolution of cooperation and extends existing work in the design of resource allocation strategies on cooperative agents. A framework for shared task processing on a network is presented, and theoretical results show sufficient conditions on distributed and asynchronous agent behaviors that guarantee an optimal allocation of task-processing resources on the network. The framework is shown to be applicable for autonomous air vehicles (AAV), mobile software agents, and smart power grids, and simulation results are given for an AAV case.

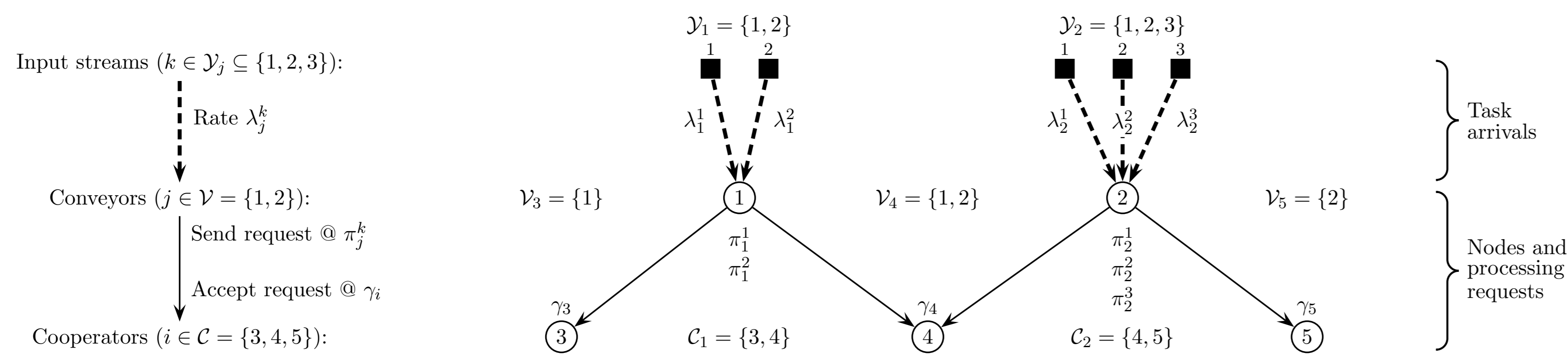


Fig. 1: Sample TPN: A flexible manufacturing system (FMS).

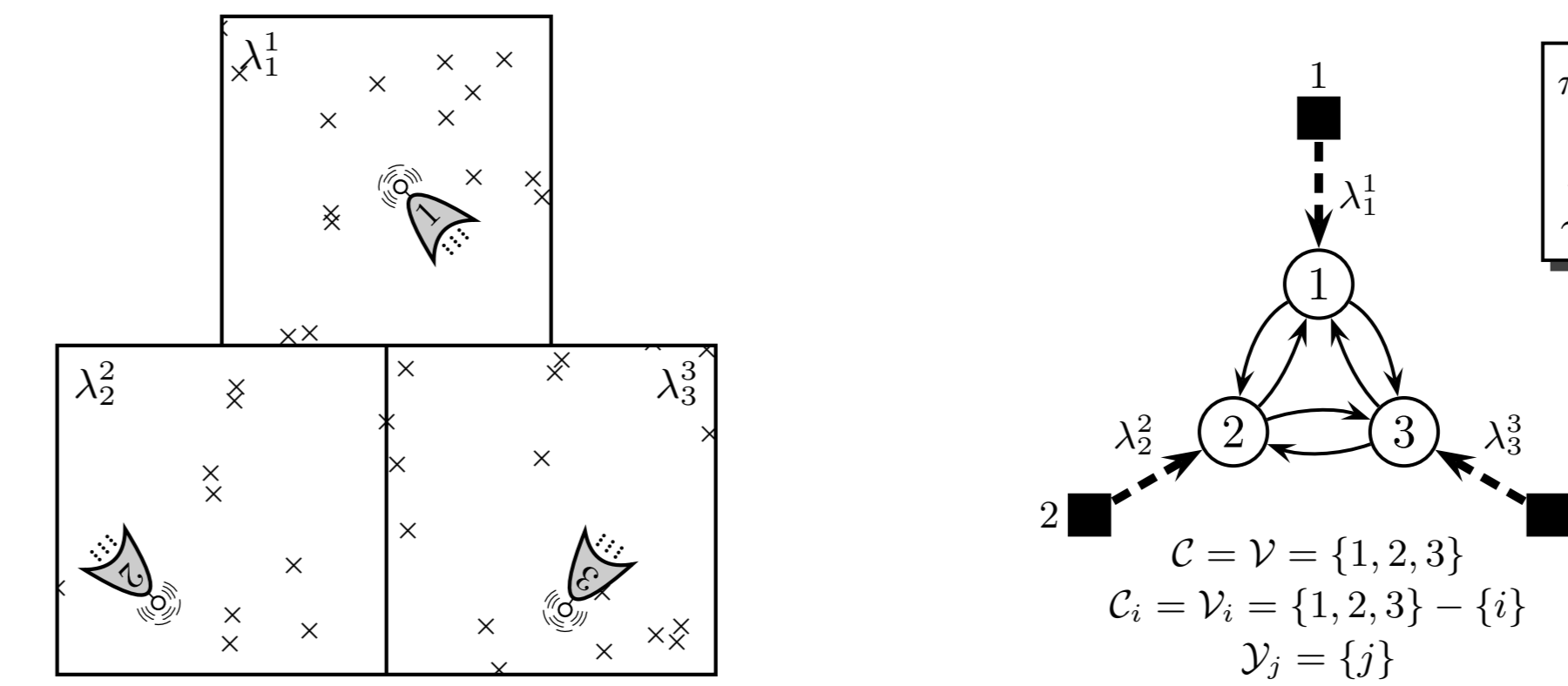


Fig. 2: Sample TPN: Autonomous air vehicle (AAV) task scheduling.

Task-processing network

- $\mathcal{A} \subset \mathbb{N}$: Set of *task-processing agents*
- $\mathcal{P} \subseteq \{(i, j) \in \mathcal{A}^2 : i \neq j\}$: Directed arcs connecting distinct agents
- $\mathcal{V}_i \triangleq \{j \in \mathcal{A} : (j, i) \in \mathcal{P}\}$: Set of *conveyors* for each $i \in \mathcal{A}$
- $\mathcal{C}_i \triangleq \{j \in \mathcal{A} : (i, j) \in \mathcal{P}\}$: Set of *cooperators* for each $i \in \mathcal{A}$
- $\mathcal{V} \triangleq \{j \in \mathcal{A} : \mathcal{C}_j \neq \emptyset\}$: Set of all conveyors
- $\mathcal{C} \triangleq \{i \in \mathcal{A} : \mathcal{V}_i \neq \emptyset\}$: Set of all cooperators
- $\mathcal{Y}_i \subset \mathbb{N}$: Possibly empty set of *task types* that arrive at conveyor $i \in \mathcal{A}$
- $\lambda_j^k \in \mathbb{R}_{>0}$: Encounter rate of type- k tasks at agent $j \in \mathcal{A}$
- $\pi_j^k \in [0, 1]$: Probability that conveyor $j \in \mathcal{A}$ advertises an incoming k -type task to its connected cooperators \mathcal{C}_j
- $\gamma_i \in [0, 1]$: Probability that cooperator $i \in \mathcal{A}$ volunteers for advertised task from one of its connected conveyors \mathcal{V}_i

Agent utility function

$$U_i(\underline{\gamma}) \triangleq \underbrace{b_i + \left(1 - \prod_{j \in \mathcal{C}_i} (1 - \gamma_j)\right) r_i - Q_i p_i(Q_i)}_{\text{Conveyor part — constant with respect to } \gamma_i} + \underbrace{\Pr(\text{Volunteer from } \mathcal{C}_i | \text{Advertisement from } i)}_{\text{Pr}(i \text{ awarded task from } j | i \text{ volunteers})} + \underbrace{\gamma_i \sum_{j \in \mathcal{V}_i} (p_{ij}(Q_j) - \text{SOBP}_1(\mathcal{C}_j - \{i\}) c_{ij})}_{\text{Cooperator part — } \gamma_i \text{ and } Q_j \text{ vary with } \gamma_i}$$

where

$$b_i \triangleq \sum_{k \in \mathcal{Y}_i} \lambda_i^k (b_i^k - c_i^k),$$

$$r_i \triangleq \sum_{k \in \mathcal{Y}_i} \lambda_i^k \pi_i^k (\gamma_i^k - (b_i^k - c_i^k)),$$

$$p_i(Q_i) \triangleq \sum_{k \in \mathcal{Y}_i} \lambda_i^k \pi_i^k p_i^k(Q_i),$$

are the costs and benefits of local processing on $i \in \mathcal{V}$, and

$$c_{ij} \triangleq \sum_{k \in \mathcal{Y}_j} \lambda_j^k \pi_j^k c_{ij}^k,$$

$$p_{ij}(Q_j) \triangleq \sum_{k \in \mathcal{Y}_j} \lambda_j^k \pi_j^k q_{ij}^k p_j^k(Q_j).$$

are the costs and benefits to $i \in \mathcal{C}$ for volunteering for tasks exported from $j \in \mathcal{V}_i$.

Asynchronous convergence to cooperation

Assume that (**Payment and topological constraints**)

1. For all $i \in \mathcal{C}$ and $j \in \mathcal{V}_i$, p_{ij} is a stabilizing payment function.
2. For all $j \in \mathcal{V}$, $|\mathcal{C}_j| \leq 3$ (i.e., no conveyor can have more than 3 outgoing links to cooperators).
3. For $i \in \mathcal{C}$ and $j \in \mathcal{V}_i$, if j is a 3-conveyor, then there must be some $k \in \mathcal{V}_i$ that is a 2-conveyor.

Define $T : [0, 1]^n \mapsto [0, 1]^n$ by $T(\underline{\gamma}) \triangleq (T_1(\underline{\gamma}), T_2(\underline{\gamma}), \dots, T_n(\underline{\gamma}))$ where, for each $i \in \mathcal{C}$,

$$T_i(\underline{\gamma}) \triangleq \min\{1, \max\{0, \gamma_i + \sigma_i \nabla_i U_i(\underline{\gamma})\}\}$$

(i.e., **gradient ascent**), where

$$\frac{1}{\sigma_i} \geq 2|\mathcal{V}_i| \max_{k \in \mathcal{V}_i} |p'_{ik}(0)|$$

for all $\underline{\gamma} \in [0, 1]^n$. If (**Hamilton's rule on networks**)

$$\underbrace{\min_{j \in \mathcal{V}_i} |p'_{ij}|}_{\text{Benefit}} (|\mathcal{C}_j|) > \underbrace{\left(|\mathcal{V}_i| - \frac{1}{2}\right)}_{\text{Relatedness}} \underbrace{\max_{j \in \mathcal{V}_i} |c_{ij}|}_{\text{Cost}}, \quad \text{for all } i \in \mathcal{C},$$

then the totally asynchronous distributed iteration (TADI) sequence $\{\underline{\gamma}(t)\}$ generated with mapping T and the outdated estimate sequence $\{\underline{\gamma}^j(t)\}$ for all $i \in \mathcal{C}$ each converge to the unique Nash equilibrium of the cooperation game.

Stabilizing payment functions

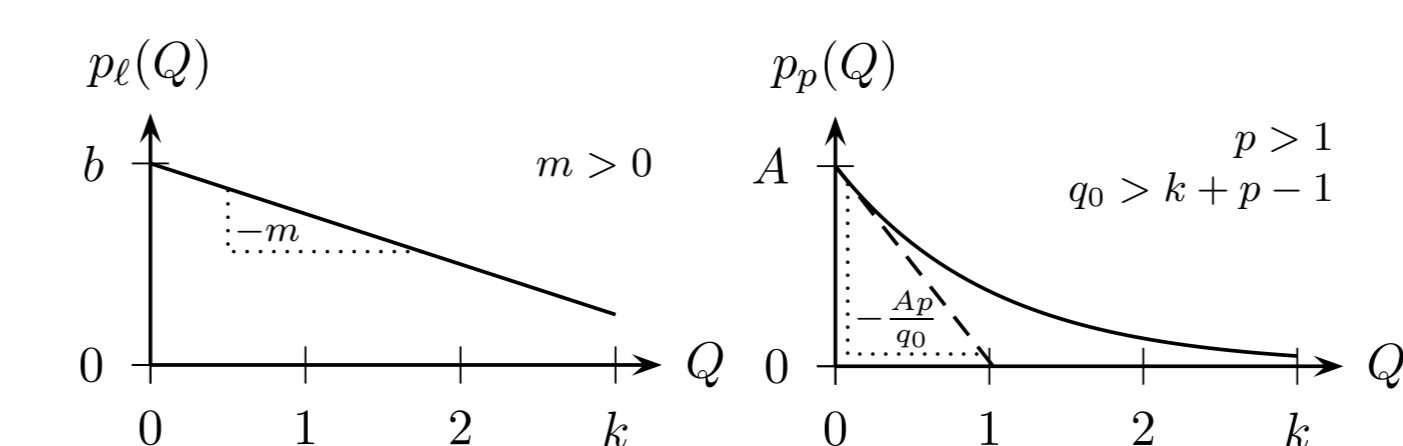


Fig. 3: Sample stabilizing payment functions

Simulation results

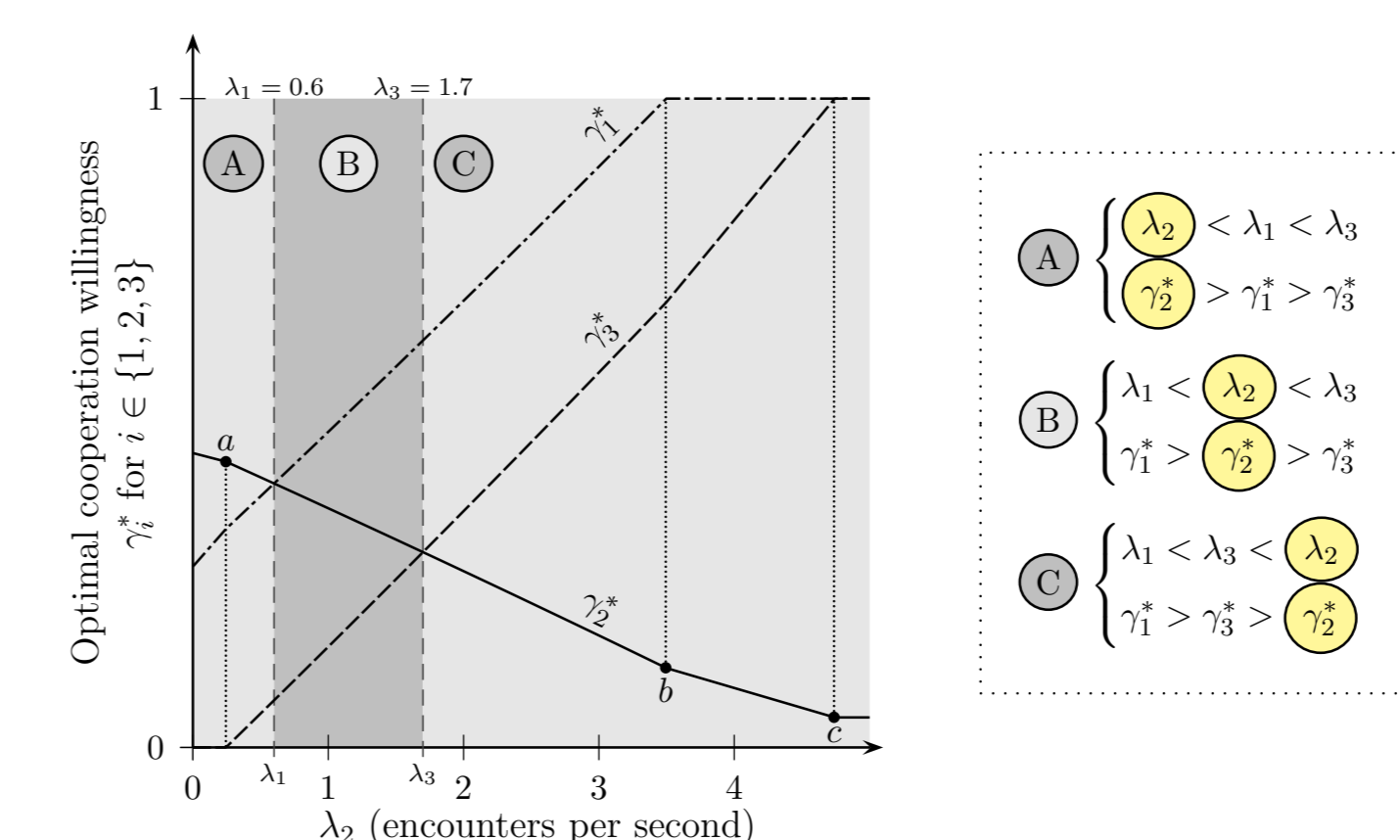


Fig. 4: Simulation of AAV patrol scenario shows convergence to predicted Nash. Increases in one encounter rate cause equilibrium shift so neighbors help more and agent helps less.

Other stable topologies

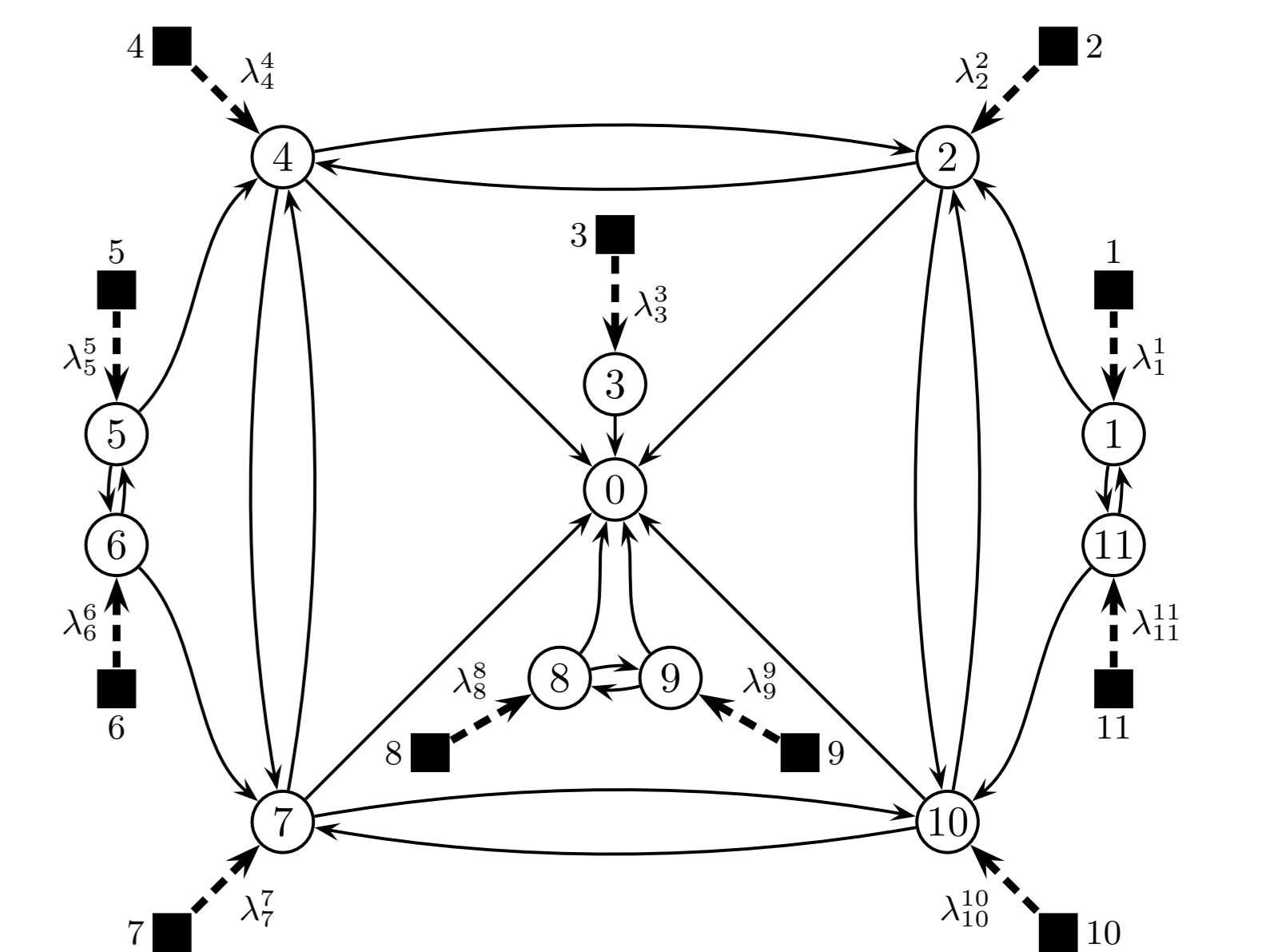


Fig. 5: Rich yet stable task-processing network. "Pills" stabilize problematic areas by focussing attention.

Acknowledgments

The germ of this paper is a doctoral candidacy research proposal. Hence, the comments of committee members Andrea Serrani, Jose B. Cruz, Jr., and Atilla Eryilmaz have helped to bring the work into its present form. The authors appreciate their expert perspectives on dynamics, game-theoretic optimization, and distributed network algorithms.